



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 56

Perfect Futures Hedge, Cross Hedge, Tailing the Hedge

J P Singh

Department of Management Studies



EXAMPLE 1

- On March 1 the spot price of a commodity is 20 and the July futures price is 19. On June 1 the spot price is 24 and the July futures price is 23.50. A company entered into a futures contracts on March 1 to hedge the purchase of the commodity on June 1. It closed out its position on June 1. What is the effective price paid by the company for the commodity?



SOLUTION 2

- **Futures Price (March 1) = 19**
- **Spot Price (June 1) = 24**
- **Futures Price (June 1) = 23.50**
- **Basis (June 1) = 0.50**
- **Effective Price = $F_0 + b_T = 19.50$**

EXAMPLE 2

- Suppose that the standard deviation of quarterly changes in the prices of a commodity is $\sigma_{\Delta S}=0.65$, the standard deviation of quarterly changes in a futures price on the commodity is $\sigma_{\Delta F}=0.81$, and the coefficient of correlation between the two changes is $\rho=0.8$. What is the optimal hedge ratio for a 3-month contract? What does it mean?



SOLUTION 1

- Hedge Ratio
- $h = \rho \sigma_{\Delta S} / \sigma_{\Delta F} = 0.80 * 0.65 / 0.81$
- $= 0.642 = Q_F / Q_S$

EFFECT OF HEDGE RATIO ON HEDGING

SPOT PRICE $t=0$	60	60	60	60	60	60
QUANTITY	100	100	100	100	100	100
SPOT PRICE $t=T$	50	60	70	50	60	70
HEDGE RATIO	0.8	0.8	0.8	1.4	1.4	1.4
FUTURES PRICE $t=0$	62	62	62	62	62	62
FUTURES PRICE $t=T$	52	62	72	52	62	72
FUTURES QUANTITY	80	80	80	140	140	140
CHANGE IN SPOT VALUE	-1000	0	1000	-1000	0	1000
CHANGE IN FUTURES VALUE	-800	0	800	-1400	0	1400
NET CHANGE	-200	0	200	400	0	-400

PERFECT HEDGING WITH FUTURES

$$\begin{aligned}\left(\sigma_{\Delta I}^2\right)_{\min} &= Q_S^2 \sigma_{\Delta S}^2 + \left(\rho \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} Q_S\right)^2 \sigma_{\Delta F}^2 - 2\rho Q_S \left(\rho \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} Q_S\right) \sigma_{\Delta S} \sigma_{\Delta F} \\ &= (1 - \rho^2) Q_S^2 \sigma_{\Delta S}^2\end{aligned}$$

Since, $\sigma_{\Delta I}^2 \geq 0$ always, $\left(\sigma_{\Delta I}^2\right)_{\min} = 0$ at best, whence $\rho = \pm 1$ for perfect hedge.



CONDITION FOR A PERFECT FUTURES HEDGE

- For a perfect futures hedge, we must have:
- $\rho = +1$ or -1 .
- Now, $\rho_{xy} = \pm 1$ is only possible if $y_i = kx_i$ holds for every observation i with the same k .
- If $\rho = +1$, and $\Delta F = k\Delta S$ then hedge ratio $Q_f/Q_s = h = \sigma_s/\sigma_f = 1/k$



- If $\rho=+1$, but the amplitudes of fluctuations of the hedged and hedging asset are not equal, we can scale the quantity (by using a hedge ratio of $1/k$, if $\Delta F=k\Delta S$) in the futures market to account for the difference in fluctuation amplitudes between the exposed asset and the hedging asset. Thus, we can still design a perfect hedge. The same holds for $\rho=-1$.
- If $\rho= +1$, the hedging position in the futures market will be opposite to the exposure and if $\rho= -1$, the hedging position will be same as the exposure.

- For ρ having some other value e.g. 0.80, we have $Q_f/Q_s = h = 0.80\sigma_s/\sigma_f$.
- We, thus, find that since only 80% of the fluctuations are inter se correlated, only 80% of the risk of the exposure ($0.80\sigma_s$) is covered by the hedge. Accordingly, the hedging quantity is calculated with reference to this 80%.
- The remaining 20% being unrelated to the hedging instrument continues to subsist in the overall hedged position. This is the reason that in such cases, the hedge is not perfect.



A CAVEAT

- Hedging invariably relates to the future evolution of prices.
- Hence, the input data for σ, ρ must also relate to future price.
- This is, obviously, not possible and the input data is usually premised on past prices. The implied assumption is that past patterns will replicate themselves.
- As such, actual outcomes of hedges may not be perfect despite the most optimal input & designing.

CROSS HEDGING

- The asset that gives rise to the hedger's exposure is sometimes different from the asset underlying the futures contract that is used for hedging.
- This is known as cross hedging.
- It leads to an increase in basis risk.



- Define S_T^* as the price of the asset underlying the futures contract at time $t=T$.
- As before, S_T is the price of the asset being hedged at time $t=T$.
- By hedging, a company ensures that the price that will be paid (or received) for the asset is $S_T + F_0^* - F_T^* = F_0^* + (S_T^* - F_T^*) + (S_T - S_T^*)$.



- The terms $S_T^* - F_T^*$ and $S_T - S_T^*$ represent the two components of the basis.
- The $S_T^* - F_T^*$ term is the basis that would exist if the asset being hedged were the same as the asset underlying the futures contract.
- The $S_T - S_T^*$ term is the basis arising from the difference between the two assets.

TAILING THE HEDGE

- Tailing adjustment is required because futures contracts give rise to mark to market settlement, thereby requiring financing of cash outflows or reinvestment of cash inflows.
- It involves a reduction in the size of the futures position relative to cash because futures prices move more than cash prices and marking to market translates these price changes into mismatched cash flows.

$$F = Se^{rT}; \Delta F = e^{rT} \Delta S > \Delta S$$

- Consider the hedging of one unit of a commodity with hedge ratio h . Then, position taken in futures market = h units since hedge ratio $h = Q_f/Q_s$.
- On minimizing the variance of $\Delta V = \Delta S + h\Delta F$, we get $h = \rho\sigma_s/\sigma_f$.

- **Let interest rates be non-stochastic.**
- Let d_t be the daily interest rate, compounded daily, applicable for day t to day $(t+1)$. Then the future value of one money unit **invested at time t** at the maturity date of futures (T) and the end of the hedge period (H) will be respectively:

$$\begin{aligned}
 FV_{t,T} &= (1 + d_t)(1 + d_{t+1})(1 + d_{t+2}) \dots (1 + d_{T-1}) \\
 FV_{t,H} &= (1 + d_t)(1 + d_{t+1})(1 + d_{t+2}) \dots (1 + d_{H-1}) \\
 \frac{FV_{t,T}}{FV_{t,H}} &= (1 + d_H)(1 + d_{H+1}) \dots (1 + d_{T-1}) = FV_{H,T}
 \end{aligned}$$

- Let us assume that we use a variable hedge ratio to be set at the beginning of each period as:
- $h_t = -1/(FV_{t+1,T})$ i.e. $h_t = -1/\left[(1+d_{t+1})(1+d_{t+2})\dots(1+d_{T-1})\right]$
- The MTM transfer at end of day 1 $\pi_0 = h_0(F_1 - F_0)$
- Since this would be invested at $t=1$, its future value at the end of hedge period $h_0(F_1 - F_0)FV_{1,H}$
- Using $h_0 = -1/(FV_{1,T})$: $-(F_1 - F_0) \frac{FV_{1,H}}{FV_{1,T}} = -(F_1 - F_0) \frac{1}{FV_{H,T}}$

- **Summing this over the entire life of the hedge:**

$$\begin{aligned}\Pi &= -(F_1 - F_0) \frac{1}{FV_{H,T}} - \dots - (F_H - F_{H-1}) \frac{1}{FV_{H,T}} + S_H \\ &= (F_0 - F_H) \frac{1}{FV_{H,T}} + S_H\end{aligned}$$

- **From the cost of carry relationship:**

$$\begin{aligned}F_t &= S_t (1 + d_t)(1 + d_{t+1})(1 + d_{t+2}) \dots (1 + d_{T-1}) = S_t FV_{t,T} \\ F_H &= S_H FV_{H,T}; \quad F_0 = S_0 FV_{0,T}\end{aligned}$$

From above : $F_t = S_t FV_{t,T}$; $F_H = S_H FV_{H,T}$; $F_0 = S_0 FV_{0,T}$

Hence, $\Pi = (F_0 - F_H) \frac{1}{FV_{H,T}} + S_H$

$$= (S_0 FV_{0,T} - S_H FV_{H,T}) \frac{1}{FV_{H,T}} + S_H = \frac{S_0 FV_{0,T}}{FV_{H,T}} = S_0 FV_{0,H}$$

- Thus, if we use a dynamic hedge ratio settled at the beginning of each period given by $h_t = -1/(FV_{t+1,T})$, we are assured of a risk free return over the hedged period, notwithstanding the MTM cash flows in the futures contract.